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Mixture Design of Experiments as Strategy for Portfolio Optimization

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ABSTRACT. Portfolio analysis is widely used by financial investors to find portfolios producing efficient results under various economic conditions. Markowitz started the portfolio optimization approach through mean-variance, whose objective is to minimize risk and maximize the return. This study is called Markowitz Mean-Variance Theory (MVP). An optimal portfolio has a good return and low risk, in addition to being well diversified. In this paper, we proposed a methodology for obtaining an optimal portfolio with the highest expected return and the lowest risk. This methodology uses Mixture Design of Experiments (MDE) as a strategy for building non-linear models of risk and return in portfolio optimization; computational replicas in MDE to capture dynamical evolution of series; Shannon entropy index to handle better portfolio diversification; and desirability function to optimize multiple variables, leading to the maximum expected return and lowest risk. To illustrate this proposal, some time series were simulated by ARMA-GARCH models. The result is compared to the efficient frontier generated by the traditional theory of Markowitz Mean-Variance (MVP). The results show that this methodology facilitates decision making, since the portfolio is obtained in the non-dominated region, in a unique combination. The advantage of using the proposed method is that the replicas improve the model precision.

Keywords: portfolio optimization; computational replicas; desirability.

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Introduction

In order to find efficient portfolios, many financial investors use portfolio theory. According to Awerbuch and Berger (2003), an efficient portfolio is a combination of investments that maximizes expected return while minimizing risk.

Many authors have searched for an optimal portfolio. Among them, we have the work of Markowitz, who defined a model of mean-variance whose objective is to minimize the risk and maximize the return (Bradri, Jadid, Rachidinejad and Moghaddam, 2008). Two other studies have attempted to minimize the variance, by estimating the covariance matrix Jagannathan and Ma (2003) and use an estimator for the covariance matrix Letoit and Wolf (2004). DeMiguel, Garlappi, Nogales and Uppal (2009) present a study whose objective is to introduce a restriction of a vector weight of the portfolio standard and, Lai, Yu and Wang (2006) used genetic algorithms with multi-objective optimization in obtaining an optimal portfolio. A hybrid heuristic approach combining multi-objective evolutionary and problem-specific local search methods is introduced by Schlottman and Seese Schlottman and Seese (2004), and Najafi and Mushakhian (2015) used a hybrid of genetic algorithm (GA) and particle swarm optimization (PSO) combining the expected value, semivariance and Conditional Value-at-Risk (CVaR) to a specified confidence level. A method based on coordinate-wise descent algorithms to optimized portfolios, in which asset weights are constrained by $1 \leq w \leq 2$ was developed by Yen and Yen (2014). We can also find an optimal portfolio using Computer Design of Experiments in Oliveira, Paiva, Lima and Balestrassi (2011). The advantage in using experimental design is the ability to gather information about a process through systematic planned experiments, in a decision model approach where the analyst could check how much a variable impact on a rated model (Leme, Paiva, Santos, Balestrassi and Galvão, 2014). In portfolio analysis, the most suitable approach to be considered is the mixture design of experiments (MDE). In this kind of experimental strategy, design factors are treated as proportions in a mixture system considered quite adequate for treating portfolios in general. The relationship of the return and risk portfolio is then estimated by an econometric model, through the response surface methodology.

However, computational experiments are deterministic, sometimes turning the issues associated with the selection of appropriate experimental designs into something different from the physical experiments. In modeling computer experiments, one is usually interested in capturing and forecasting the complex behavior of responses under analysis, as well as in avoiding lower dimension projection problems. Problems, such as variance reduction, blocking and randomization are usually not considered. Furthermore, in computer experiments, the residual between the observed and fitted values is not a stochastic error, but a model bias. However, the central limit theorem, as the bias may be considered as a sum of a number of multiple higher-order small quantities.

These issues gave rise to many alternative designs for computer experiments, such as space-filling designs. In this approach, one spans experimental points over a design region, enabling to capture complex response behavior and/or mimicking statistical properties of the system under analysis. Researchers have adopted different approaches for space-filling designs (Pronzato and Muller, 2012). However, for experiments with mixtures, MDE is the optimal strategy to space-fill the feasible region (Cornell, 2002).

The objective of this study was to find an optimal portfolio through a Mixture Design of Experiments (MDE) using computational replicas. To emulate random errors, in each replica the forecasting of the series is used. The advantage of using the proposed method is that replicas improve the model precision and this is important when we are working with forecasting and gives reliability for decision-making.

This article is organized as follows: Section 2 presents fundamental concepts of portfolio optimization, mainly on the theory of Markowitz Mean-Variance (MVP). Section 3 presents the concepts of optimization based on Mixture Design of Experiments. The Shannon entropy index, the concepts of replicas, and desirability functions are also covered. In section 3 also presents an application of the proposed methodology, where time series are simulated and an optimal portfolio is obtained. Finally, the last section presents some concluding remarks.

Material and methods

Using mixture design of experiments for portfolio optimization

In this section, we present basic concepts of portfolio optimization, Shannon entropy index, replicates in Mixture Design of Experiments (MDE), and the desirability function. These concepts are presented in Monticeli, Balestrassi, Souza, Leme and Paiva (2017).

Mixture design of experiments for portfolio

Based on return and risk analysis, portfolio theory provides information to assist investors in making decisions. According to Monticeli et al. (2017), to make this process more efficient, some researchers have developed portfolio optimization models. The investor's goal is to maximize return and minimize risk. The investor must consider the factors: the expected return, the risk and the amount invested in each asset (Oliveira et al., 2011).

In 1952, Harry Markowitz (1952) presented a study called Mean-Variance (MVP). MVP consists of creating a portfolio with minimum variance and maximum expected return. The expected return is defined in (1), i.e., is the sum of the weighted average of each asset in the proportions of investment.

$$\mu_c = \sum_i^n w_i \mu_i. \quad (1)$$

The variance defined by Markowitz is presented in (2), written as the expected value of the linear combination involving a covariance term σ_{ij}.

$$\sigma^2 = \sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{j \neq i} w_i w_j \sigma_{ij}. \quad (2)$$

The MVP can be characterized as an MDE if considering the weights and values of the MVP model as proportions of a mixture, whose sum is a unity, or restricted to a certain limit.

Cornell defines the MDE as a special class of response surface experiments, in which the product under investigation consists of multiple components. The response to be obtained is a function of the proportions of ingredients that comprise the mixture. These proportions are non-negative and the sum must be equal to one (Cornell, 2002). The components of the mixture experiment form a space called simplex coordinate system; the uniform distribution is known as a lattice.

The lattice can have a match with a polynomial equation. A polynomial model of degree m to a mixture of q components consists of a coordinated set of points that define the proportions of each component, according to Equation (3). This model is called $\{q, m\}$simplex-lattice (Cornell, 2002).

$$w_i = 0, \frac{1}{m}, \frac{2}{m}, \dots, 1 \quad (3)$$

Figure 1 illustrates a {3,2} simplex-lattice. In this case, the mixture has three components and the polynomial is of degree two. In Equation (4), we have the points that form the lattice. The first three components are the nodes and represent the pure mixture, the last three terms represent mixtures of two components (Oliveira et al., 2011).

$$(w_1, w_2, w_3) = (1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right), \left(0, \frac{1}{2}, \frac{1}{2}\right). \quad (4)$$

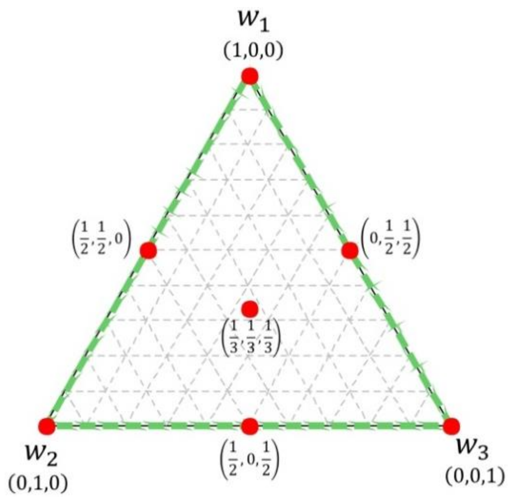

Figure 1: Simplex coordinate system with three components.

In Figure 1, we can also observe a central point, called centroid. This point is the mixture with equal proportions of each component (Oliveira et al., 2011).

The polynomial model of degree m that represents the relationship between the response variables and the proportions, is generally linear, quadratic or cubic. Equation (5) shows an example of a cubic polynomial model.

$$E(w) = \sum_{i=1}^{q} \beta_i^* w_i + \sum\sum_{i<j}^{q} \beta_{ij}^* w_i w_j + \sum\sum_{i<j<k}\sum^{q} \beta_{ijk}^* w_i w_j w_k \quad (5)$$

The coefficients β_i^* show how each component contributes to the response variable. In the same way, the term β_{ij}^* indicates the combined effect of the components i and j. Indeed, for the linear model $\beta_{ij}^* = \beta_0 + \beta_i$ and for the quadratic model, $\beta_i^* = \beta_0 + \beta_i + \beta_{ii}$ and $\beta_{ij}^* = \beta_{ij} - \beta_{ii} - \beta_{ij}$. These coefficients are estimated using the Ordinary Least Squares algorithm (OLS) [13].

MDE can generate concentrated portfolios. To avoid it, we can use a maximization of the Shannon entropy index, as presented in Monticeli et al. (2017).

Shannon entropy

Portfolio diversification consists of distributing the weights among the assets and not attribute most of the weight to just a few assets, which would increase the risk. According to [16], a way to control diversity is by restricting weights, imposing an upper or lower limit. In this section, these restrictions are defined as proposed by Usta and Kantar (2011).

Finding an optimal portfolio is to maximize the objective function, $f(w_i)$, subject to the constrained presented in (6) and (7).

$$0 \le w_i \le 1, \quad for\ i = 1\ to\ n \quad (6)$$

$$\sum_{i=1}^{n} w_i = 1 \quad (7)$$

Adding to the constraints (6) and (7) an entropy constraint, whose objective is to add a lower limit to the weights in portfolio, Shannon (1948) defines entropy as a discrete set of probabilities $p_1, p_2, \dots, p_n$ as in Equation (8).

$$Ent = -\sum_{i=1}^{n} p_i(x) \log p_i(x) \quad (8)$$

For a continuous distribution with a density distribution $p(x)$, Shannon (1952), defines entropy according to Equation (9).

$$Ent = -\int_{-\infty}^{\infty} p(x) \log p(x) dx \quad (9)$$

Note that restriction (6), which ensures non-negative weights, meets the necessary condition allowing the calculation of entropy.

The entropy index in a portfolio has a discrete probability distribution. Therefore, as defined in Huang (2012), the entropy constraint adds a lower limit L_E on the entropy Ent of a portfolio w_i, according to Equation (10).

$$Ent = -\sum_{i=1}^{n} p_i(x) \log p_i(x) = -\sum_{i=1}^{n} w_i \log w_i \geq L_E \quad (10)$$

When the entropy index reaches the minimum value, there is a less diversified scenario with only one component of w is 1, because, $-1 \times \log 1 = 0$. In the most diverse scenario, $w_i = \frac{1}{n}$ for all i, Ent reaches its maximum $-n\left(\frac{1}{n}\log\frac{1}{n}\right) = \log n$. Thus, the range of L_E is $[0, \log n]$. Since a larger Ent indicates better diversity, the entropy constraint uses a lower limit L_E within the interval $[0, \log n]$ to control the diversity of w_i from being too low (Lin, 2013).

Replicas in MDE and Desirability function

In MDE, replicas are identical experiments but with different characteristics. It is possible to replicate all the points of a project, which may provide a better estimate of the error or noise, favoring more accurate estimates of the effects.

In this study, an optimal portfolio will be obtained using the entropy index to diversify it and replica to better estimate the errors. To solve the optimization of multiple responses, we used the desirability function.

The desirability function transforms each estimated response variable $\hat{y}_i$ into a desirable individual value d_i, where $0 \leq d_i \leq 1$. When it is desired to minimize the response variable y, the function of transforming variables is given by Equation (11). When it is desired to maximize the response variable y, Equation (12) is used for the variable transformation function.

$$d[y] = \begin{cases} 0 & if \quad \hat{y}_i > H_i \\ \left[\frac{(H_i - \hat{y}_i)}{(H_i - T_i)}\right]^{\lambda} & if \quad T_i \leq \hat{y}_i \leq H_i \\ 1 & if \quad \hat{y}_i < T_i \end{cases} \quad (11)$$

$$d[y] = \begin{cases} 0 & if \quad \hat{y}_i < L_i \\ \left[\frac{(\hat{y}_i - L_i)}{(T_i - L_i)}\right]^{\lambda} & if \quad L_i \leq \hat{y}_i \leq T_i \\ 1 & if \quad \hat{y}_i > T_i \end{cases} \quad (12)$$

where, L_i is the lower limit of desirability; H_i is the upper limit; T_i is the target of the desirability, and λ is the parameter of desirability; when $\lambda \sim 1$, equal emphasis is given to the target and limits; when $\lambda \sim 10$, $\hat{y}_i$ assumes a value closer to the target (Oliveira et al., 2011). In Figure 2, one can observe how the weights affect the result.

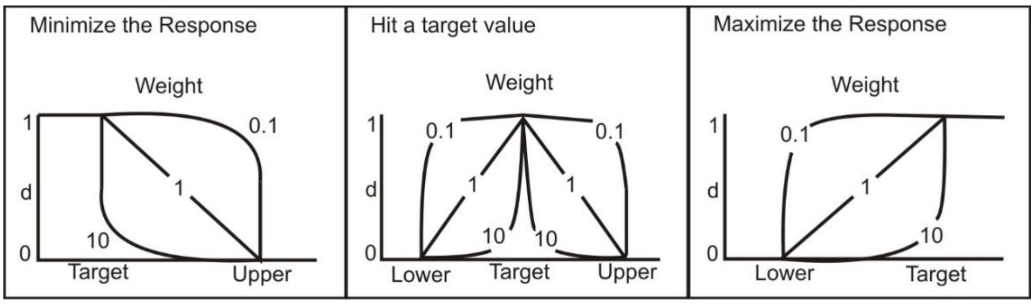

Figure 2. Desirability functions for different goals - how weights affect their shapes.

The total desirability value D, which will be in the interval $[0, 1]$, is formed by the simple geometric mean as in Equation (13) or by the weight geometric mean given by Equation (14), in this case, the weights (z_i) indicate the importance of each property in relation to others in the optimization.

$$D = (d_1 \times d_2 \times \dots \times d_k)^{\frac{1}{k}} \quad (13)$$

$$D = \left(d_1^{z_1} \times d_2^{z_2} \times \dots \times d_k^{z_k}\right)^{\frac{1}{\sum_i^k z_i}} \quad (14)$$

where k is the number of variables.

Considering an MVP, we can reformulate the portfolio optimization problem using the desirability function in MDE, as in Oliveira et al. (2011), where the mean, μ_c, is the return, the variance, σ^2, the risk measure, and seeking to portfolio diversification, the entropy, Ent, according to Equation (15).

$$\begin{aligned} &Max \quad D = \sqrt{d_{\mu_c} \times d_{\sigma^2} \times d_{Ent}} \\ &subject\ to: \quad d^{n+1}(y_i) \geq D, \quad i = 1, 2, \dots, k \\ &\quad D \geq 0 \\ &\quad w \in \Omega \end{aligned} \quad (15)$$

$d^{n+1}(y_i)$ is the desirability function y_i in $(n+1)^{th}$iteration; $w \in \Omega$ denotes the entire region defined early in the process.

Results and discussion

Portfolio Optimization by MDE and computational replicas: a case study on ARMA-GARCH times series

In this section, we apply the proposed methodology for portfolio optimization. The portfolio consisted of time series generated from the model chosen a priori.

The returns and risks are defined as 2.1, that is, the average return of portfolio p, (denoted by $avret_p$), is defined by Equation (16)

$$avret_p = \sum_i w_i . \mu_i \quad (16)$$

In which, w_i is the weight of series i in portfolio p. The risk of the portfolio is defined by Equation (17), with σ_i^2 the variance of the series i e σ_{ij} is correlation between series i and j

$$\sigma_p = \sqrt{\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{j \neq i} w_i w_j \sigma_{ij}} \quad (17)$$

The portfolio of lower cost and lower risk for w_i, can be determined by minimizing Equation (16) and Equation (17) with restrictions $\sum_i w_i = 1$ and $w_i \geq 0, \forall i \in I$.

The time series that make up the portfolio were generated following ARMA-GARCH models.

ARMA-GARCH model

We used for time series the so called ARMA(p, q)-GARCH(k, l) models. The ARMA model was employed to fit the mean, and GARCH model for variance (Montgomery, Jennings and Kulahci, 2011).

Consider the time series y_t, the autoregressive and moving average model, ARMA(p, q) can be modeled by Equation (18).

$$y_t = \delta + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j} + \varepsilon_t \quad (18)$$

where δ is a constant term, ϕ_i is the ith autoregressive coefficient, θ_j is the jth moving average coefficient, and ε_t is the error term at time t. ε_t can be modeled as Equation (19),

$$\varepsilon_t = \sqrt{v_t} z_t \quad (19)$$

where z_t is a white noise sequence with mean 0 and variance 1. Assuming that v_t is conditioned on the l previous erros and can be estimated by Equation (20)

$$v_t = \zeta_0 + \eta_1\varepsilon_{t-1}^2 + \eta_2\varepsilon_{t-2}^2 + \cdots + \eta_l\varepsilon_{t-l}^2 \quad (20)$$

where ζ_0 and η_i are constant coefficients. In this case, ε_t is said to follow an autoregressive conditional heteroskedastic process of order l, denoted by ARCH(l) (Liu, Erdem & Shi, 2011).

When the current conditional variance depends on the previous conditional variance, the GARCH(k, l) (Generalized Autoregressive Conditional Heteroscedasticity) process given by

$$v_t = \zeta_0 + \sum_{i=1}^{k}\zeta_i v_{t-i} + \sum_{i=1}^{l}\eta_i\varepsilon_{t-i}^2 \quad (21)$$

To check the best model to forecast, we used the Akaike's Information Criterion (AIC).

Akaike's Information Criterion - AIC

Akaike's Information Criterion (AIC) suggested by Akaike (1973) uses a function based on the likelihood to select the best fitted model. AIC can address the over-fitting problem by introducing the penalty term based on the number of free parameters. The mathematical model for the AIC is presented in

$$AIC = 2k - 2\ln(L) \quad (22)$$

where L and k are the values of the likelihood function and the number of free parameters, respectively. The models with lower AIC values are usually preferred (Liu et al., 2011).

The times series

We present the generated series that make up the portfolio and the models used to better model the series for the forecasts. The four time series and the model used for the forecasts are shown in Figures 3, 4, 5 and 6.

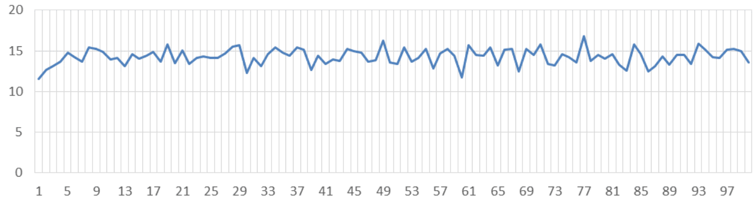

Figure 3. Serie 1, equation $y_t = 10 + 0.3y_{t-1} + \varepsilon_t - 0.7\varepsilon_{t-1}$. The model used: ARMA(1,1) with AIC: 287.19.

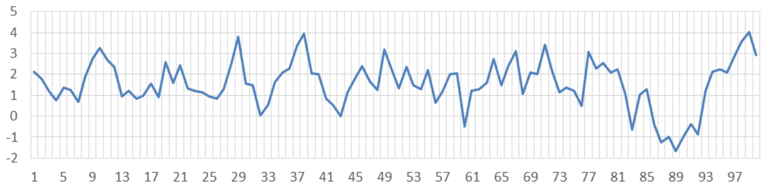

Figure 4. Serie 2, equation $y_t = 0.6 + 0.38y_{t-1} + 0.17y_{t-2} + \varepsilon_t + 0.08\varepsilon_{t-1}$. The model used: ARMA(1,0)-GARCH(1,1) with AIC: 270.906.

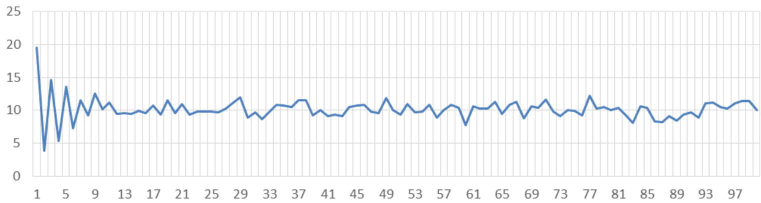

Figure 5. Serie 3 equation $y_t = 18 - 0.8y_{t-1} + \varepsilon_t + 0.8\varepsilon_{t-1}$. The model used: ARMA(1,1)-GARCH(0,1) with AIC: 338.46.

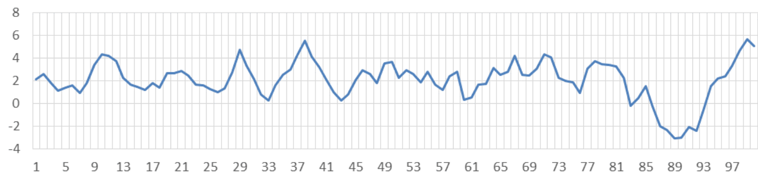

Figure 6: Serie 4, equation $y_t = 0.6 + 0.26y_{t-1} + 0.37y_{t-2} + \varepsilon_t + 0.8\varepsilon_{t-1}$. The model used: ARMA(1,1)-GARCH(1,1) with AIC: 275.778.

After selecting the model, we made the forecast of 5 steps in each series. The return and risk are presented in Table 1 and 2, respectively.

Table 1. Forecast return for each series.

	Series 1	Series 2	Series 3	Series 4
Step	14.97254	2.4129	10.2307	3.9383
1	14.78319	2.0938	10.0975	3.2420
2	14.64562	1.8995	10.2125	2.8119
3	14.54567	1.7813	10.1132	2.5463
4	14.47306	1.7093	10.1989	2.3823
5	14.97254	2.4129	10.2307	3.9383

Table 2. Forecast risk for each series.

	Series 1	Series 2	Series 3	Series 4
Step	0.969704	0.9959	0.8444	1.0523
1	0.980197	0.9969	1.1942	1.0541
2	0.985691	0.9979	1.4626	1.0559
3	0.988578	0.9989	1.6889	1.0577
4	0.990099	0.9999	1.8882	1.0595
5	0.969704	0.9959	0.8444	1.0523

For each replica, we use the forecast return and risk, as the values in Table 1 and Table 2. In this way, we have five replicas. For each replica, we calculate the correlation between the time series.

Applying the proposed methodology

After known the series that composed the portfolio, the forecasts and the correlations, we find the optimum portfolio using the proposed methodology, i.e., using MDE with replicas. Using the desirability function implemented in the Minitab software and, to improve the diversity of the portfolio, we used the Shannon entropy index.

Five replicates, one for each forecast were used. It was used the entropy to diversify the portfolio, Equation (10). The weights, w_i, of each series must not be zero. Thus, we need to delimit the proportions between 0.01% for the lower level and 99.97% to the upper level. We added points in the design, which are called axial points, in order to provide information about the interior of the response surface, which tends to improve the final result (Cornell, 2002). In Figure 7, we can see the MDE as well as the added axial point.

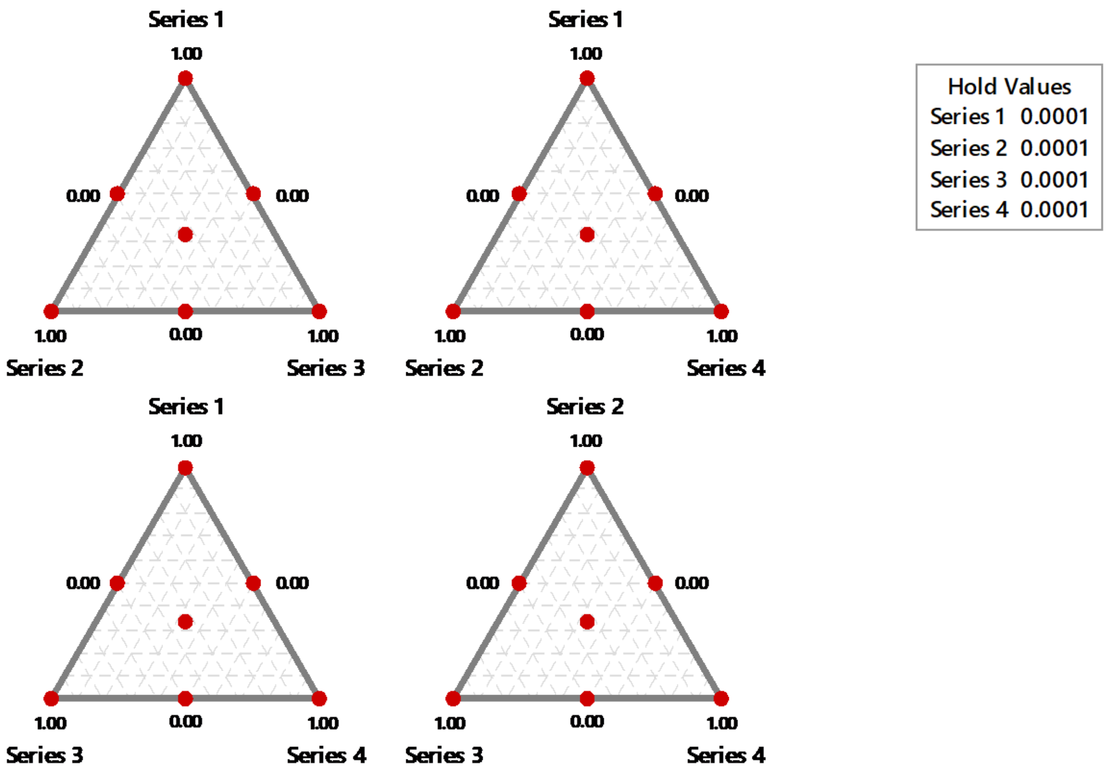

Figure 7. Mixtures arrangements of 4 time series.

Table 3 lists the constructed MDE. The return and risk were calculated using Equation (16) and Equation (17) respectively, and the entropy, using Equation (10). All taking into account the weights, w_i, obtained by the MDE.

Table 3. MDE with 4 components and 5 replicas in Minitab.

	Series 1	Series 2	Series 3	Series 4	Return	Risk	Entropy
Replica 1	0.00010	0.00010	0.00010	0.99970	3.9399	1.02565	0.00306
	0.99970	0.00010	0.00010	0.00010	14.9697	0.98455	0.00306
	0.00010	0.99970	0.00010	0.00010	2.4151	0.99784	0.00306
	0.00010	0.00010	0.99970	0.00010	10.2298	0.91876	0.00306
	0.00010	0.00010	0.49990	0.49990	7.0848	0.79124	0.69505
	0.00010	0.49990	0.00010	0.49990	3.1775	0.98623	0.69505
	0.00010	0.49990	0.49990	0.00010	6.3224	0.83405	0.69505
	0.49990	0.00010	0.00010	0.49990	9.4548	0.78698	0.69505
	0.49990	0.00010	0.49990	0.00010	12.5997	0.79035	0.69505
	0.49990	0.49990	0.00010	0.00010	8.6924	0.86688	0.69505
	0.25000	0.25000	0.25000	0.25000	7.8886	0.76800	1.38629
	0.12505	0.12505	0.12505	0.62485	5.9142	0.85067	1.07378
	0.62485	0.12505	0.12505	0.12505	11.4292	0.80457	1.07378
	0.12505	0.62485	0.12505	0.12505	5.1519	0.87178	1.07378
	0.12505	0.12505	0.62485	0.12505	9.0592	0.77702	1.07378
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Replica 5	0.00010	0.00010	0.00010	0.99970	2.3842	1.02915	0.00306
	0.99970	0.00010	0.00010	0.00010	14.4701	0.99485	0.00306
	0.00010	0.99970	0.00010	0.00010	1.7115	0.99984	0.00306
	0.00010	0.00010	0.99970	0.00010	10.1977	1.37379	0.00306
	0.00010	0.00010	0.49990	0.49990	6.2910	0.94237	0.69505
	0.00010	0.49990	0.00010	0.49990	2.0479	0.98764	0.69505
	0.00010	0.49990	0.49990	0.00010	5.9546	0.97828	0.69505
	0.49990	0.00010	0.00010	0.49990	8.4272	0.79273	0.69505
	0.49990	0.00010	0.49990	0.00010	12.3339	0.94346	0.69505
	0.49990	0.49990	0.00010	0.00010	8.0908	0.87090	0.69505
	0.25000	0.25000	0.25000	0.25000	7.1909	0.81080	1.38629
	0.12505	0.12505	0.12505	0.62485	4.7876	0.86234	1.07378
	0.62485	0.12505	0.12505	0.12505	10.8305	0.82009	1.07378
	0.12505	0.62485	0.12505	0.12505	4.4512	0.88234	1.07378
	0.12505	0.12505	0.62485	0.12505	8.6943	1.00573	1.07378

Using Equation (5), response surfaces were built for each portfolio property using the OLS algorithm, leading to the following objective functions:

$$E(\mu) = 14.6840w_1 + 1.9794w_2 + 10.1706w_3 + 2.9842w_4$$

$$E(\sigma^2) = 0.9913w_1 + 0.9996w_2 + 1.1816w_3 + 1.0286w_4 - 0.5171w_1w_2 - 0.8466w_1w_3 - 0.8892w_1w_4 - 0.7072w_2w_3 - 0.1169w_2w_4 - 0.9193w_3w_4 \quad (23)$$

$$E(Ent) = -0.03097w_1 - 0.03097w_2 - 0.03097w_3 - 0.03097w_4 + 3.41042w_1w_2 + 3.41042w_1w_3 + 3.41042w_1w_4 + 3.41042w_2w_3 + 3.41042w_2w_4 + 3.41042w_3w_4$$

To determine the optimal portfolio by the desirability function, it is necessary to define the parameters of Target (T_i), Upper Limit (H_i), and Lower Limit (L_i). These parameters can be defined using the average, maximum value and minimum value for return, risk and entropy.

Since the purpose of the risk is to minimize, the value used were: for the Target (T_i), the minimum value and the Upper Limit (H_i), the average. For return and entropy, as the objective is to maximize, values used were: for the Target (T_i), the maximum value and for the Lower Limit (L_i), the average. Table 4 lists the values used.

Table 4. Parameters used to maximize the desirability function.

	Return	Risk	Entropy
T_i	14.9697	0.7680	1.3863
L_i	7.4545		0.6576
H_i		0.9170	

From these parameters, and the desirability function used in Minitab, we obtained the following combination for the portfolio, listed in Table 5.

Table 5. Obtained by combining desirability function.

Series 1	Series 2	Series 3	Series 4
48.48%	5.08%	32.30%	14.15%

The value of return of the optimal portfolio found (Table 5) is 10.9258, and the risk is 0.7978, as noted in Figure 8. Figure 9 illustrates the mixture contour plot for returns of portfolio.

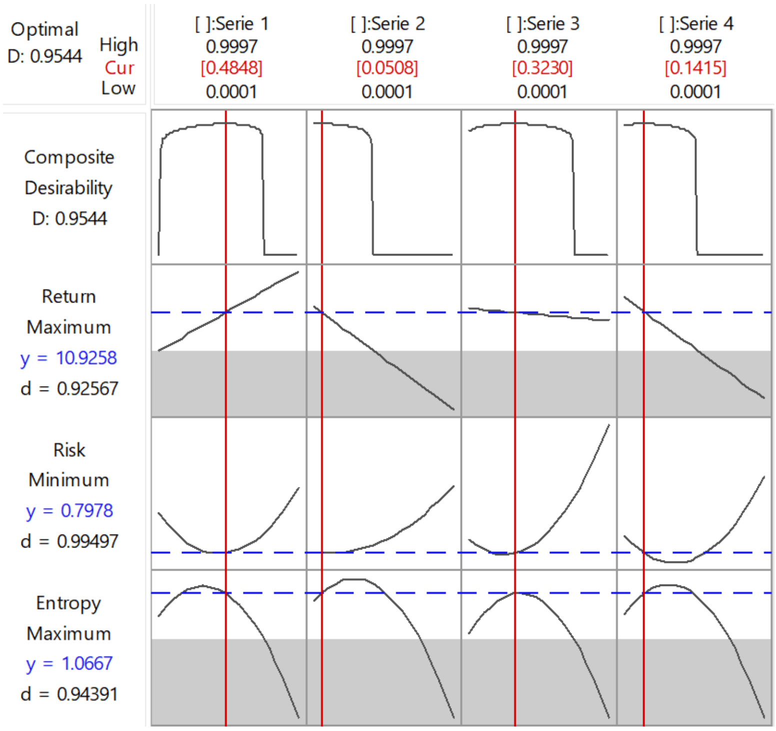

Figure 8. Optimal portfolio obtained by the desirability function.

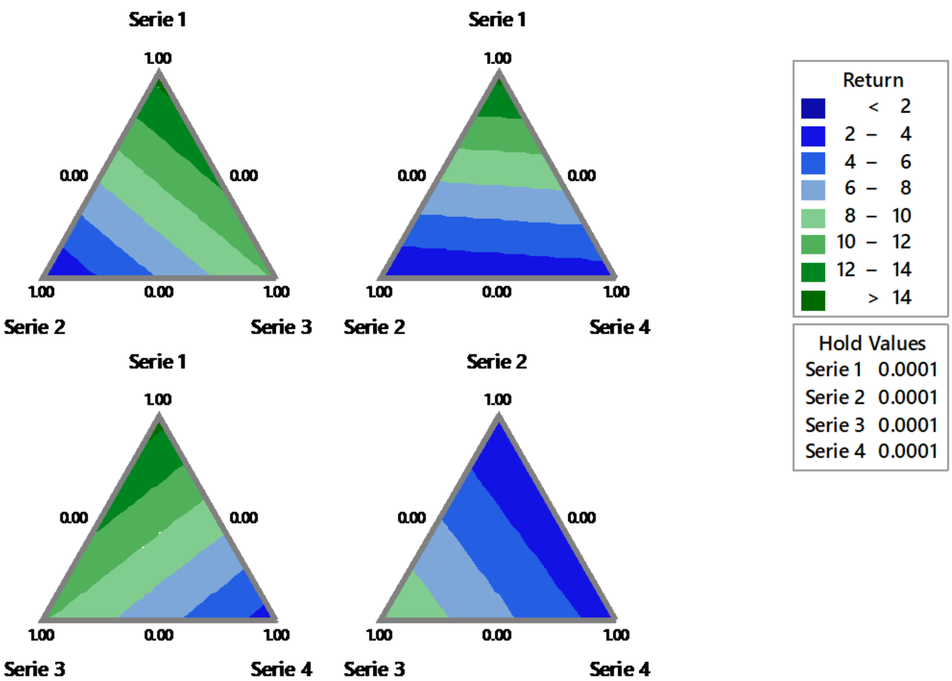

Figure 9. Contour plot for the Portfolio's return.

Analyzing the sensitivity of each set, we have: in series 1, we get the values that are in the following ranges: from 0. 88% to 75.57%. In series 2, values within the range 0.01% to 41.90%. For series 3, we have the values between 0.01% and 75.17%, and for series 4, 0.01% and 51.66%. Table 6 lists some results obtained by varying the value of series 2.

Table 6. Variation of weights applied in series 2.

Series 1	Series 2	Series 3	Series 4	Desirability	Return	Risk	Entropy
51.06%	0.01%	34.02%	14.90%	0.9530	11.4037	0.8002	1.0033
49.54%	3.00%	33.01%	14.46%	0.9541	11.1218	0.7986	1.0422
48.48%	5.08%	32.30%	14.15%	0.9544	10.9258	0.7978	1.0667
45.96%	10.00%	30.62%	13.41%	0.9535	10.4620	0.7967	1.1164
40.85%	20.00%	27.22%	11.92%	0.9457	9.5194	0.7984	1.1814
35.75%	30.00%	23.82%	10.43%	0.9273	8.5769	0.8053	1.1983
30.64%	40.00%	20.42%	8.94%	0.8701	7.6343	0.8174	1.1674

We can also point out a strong negative correlation between the risk and the Entropy (-0.709), i. e., the higher the entropy, the lower the risk. Therefore, by maximizing the entropy, the risk is minimized.

To validate the proposed methodology, we solved the problem using the theory of Markowitz Mean-Variance and traced the efficient frontier (Figure 10). Then, we marked in the same coordinate plane the optimal portfolio obtained by the proposed methodology ($Point\ A$).

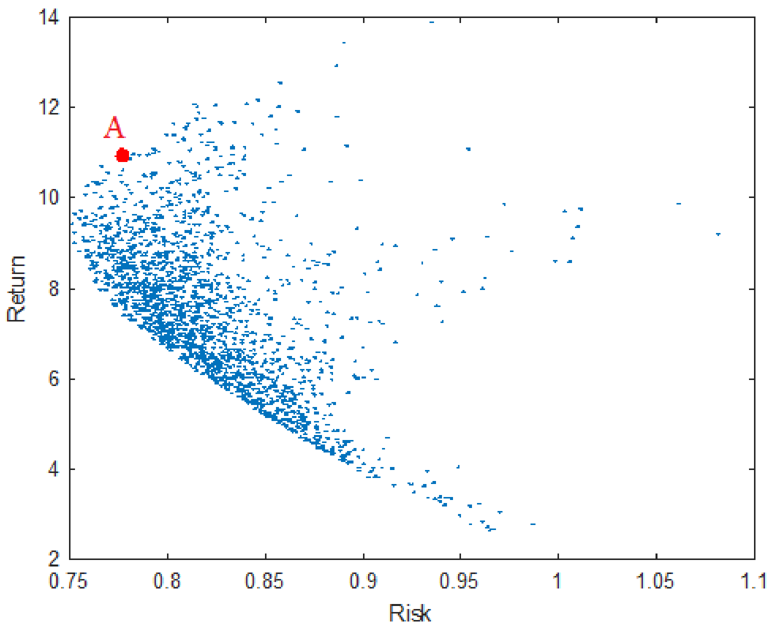

Figure 10: Efficient frontier determined by MVP.

Comparing the results obtained with no use of replicas, we get an increase of 5.34% in return and a reduction of 0.81% in risk. Also, we have reduced the standard deviation of returns and risk at 15.66% and 27.26%, respectively.

$Point\ A$ in Figure 10 is in the frontier, located in the combination of more return and lower risk, which allows to conclude that the result was satisfactory. It is worth noting that, as the proposed methodology determines an optimal combination for the portfolio, it is easier to make a decision, which does not occur with MVP, as it generates a set of optimal combinations.

The advantage of the proposed methodology compared to that presented by Monticeli et al. (2017) is related to the number of replicates. The number of replicas used in this methodology is defined by the number of forecasts made, while in the methodology presented in Monticeli et al. (2017), the number of replicas is related to the autocorrelation function and the number of moving windows, which can result in a very large number of replicas.

Conclusion

This study presented a methodology using Mixture Design of Experiments to determine the combination for an efficient portfolio. In the methodology used, to come up with a diversified portfolio, we can use the Shannon entropy index, and for multi-objective optimization, the desirability function. We use replicas in Mixture Design of Experiments in the forecasts to provide more security to decision making. The work also compared its results with the traditional theory of Markowitz Mean-Variance.

In the proposed method, the result is an optimal combination of lower cost and lower risk, which facilitates the decision-making process on portfolio selection.

References

Akaike, H. (1973). Information theory and extension of the maximum likelihood principle. In B. N. Petrov, F. Cszaki (Eds.), *Second International Symposium on Information Theory* (p. 267-281). Budapest, Akademiai Kiado.

Awerbuch, S., & Berger, M. (2003). Applying portfolio theory to EU electricity planning and policy-making. *IEA/EET International Energy Agency, 3*, 69.

Bradri, A., Jadid, S., Rashidinejad, M., & Moghaddam, M. P. (2008) Optimal bidding strategies in oligopoly markets considering bilateral contracts and transmission constraints. *Electric Power Systems Research, 78*(6), 1089-1098. DOI: https://doi.org/10.1016/j.epsr.2007.09.003.

Cornell, J. A. (2002). *Experiments with mixture: Design, models and the analysis of mixture data* (3th ed., p. 680). New York: John Wiley & Sons.

DeMiguel, V., Garlappi, L., Nogales, F. J., & Uppal, R. (2009). A Generalized Approach to Portfolio Optimization: Improving Performance by Constraining Portfolio Norms. *Management Science, 55*(5), 798–812. Retrieved from http://www.jstor.org/stable/40539189.

Huang, X. X. (2012). An entropy method for diversified fuzzy portfolio selection. *International Journal of Fuzzy Systems 14*(1), 160-165. Retrieved from https://bitlybr.com/QlubK

Jagannathan, R., & Ma, T. (2003). Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps. *The Journal of Finance, 58*(4), 1651-1683. DOI: http://www.jstor.org/stable/3648224ax.

Lai, K. K., Yu, L., & Wang, S. (2006) Mean-Variance-Skewness-Kurtosis-based Portfolio Optimization. In *Computer and Computational Sciences. First International Multi-Symposiums on Computer and Computational Sciences* (p. 292-297). Hangzhou, China. DOI: 10.1109/IMSCCS.2006.239.

Leme, R. C., Paiva, A. P., Santos, P. E., Balestrassi, P. P., & Galvão, L. L. (2014). Design of experiments applied to environmental variables analysis in electricity utilities efficiency: The Brazilian case. *Energy Economics, 45*, 111–119. DOI: https://doi.org/10.1016/j.eneco.2014.06.017.

Letoit, O., & Wolf, M. (2004) A well-conditioned estimator for large-dimensional covariance matrices. *Journal of Multivariate Analysis, 88*(2), 365-411. DOI: https://doi.org/10.1016/S0047-259X(03)00096-4.

Lin, J.-L. (2013). On the diversity constraints for portfolio optimization. *Entropy, 15*(12), 4607-4621. DOI: https://doi.org/10.3390/e15114607.

Liu, H., Erdem, E., & Shi, J. (2011). Comprehensive evaluation of ARMA-GARCH(-M) approaches for modeling the mean and volatility of Wind speed. *Applied Energy, 88*(3), 724-732. DOI: https://doi.org/10.1016/j.apenergy.2010.09.028.

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance, 7*(1), 77-91. DOI: https://doi.org/10.2307/2975974

Montgomery, D., Jennings, C., & Kulahci, M. (2011). *Introduction to time series analysis and forecasting*. New Jersey: Wiley.

Monticeli, A. R., Balestrassi, P. P., de Souza, A. C. Z., Leme, R. C., & Paiva, A. P. (2017). Mixture design of experiments on portfolio optimization of power generation. *IET Generation, Transmission & Distribuition, 11*(2), 322-329. DOI: https://doi.org/10.1049/iet-gtd.2016.0422.

Najafi, A. A., & Mushakhian, S. (2015). Multi-stage stochastic mean-semivariance-CVaR portfolio optimization under transaction costs. *Applied Mathematics and Computation, 256*(1), 445-458. DOI: https://doi.org/10.1016/j.amc.2015.01.050.

Oliveira, F. A. de, Paiva, A. P., Lima, J. W. M., Balestrassi, P. P., & Mendes, R. R. A. (2011). Portfolio optimization using Mixture Design of Experiments: Scheduling trades within electricity markets. *Energy Economics, 33*(1), 24–32. DOI: https://doi.org/10.1016/j.eneco.2010.09.008.

Pronzato, L., & Muller, W. G. (2012). Design of computer experiments: space filling and beyond. *Statistics and Computing*, *22*, 681-701. DOI: https://doi.org/10.1007/s11222-011-9242-3.

Schlottman, F., & Seese, D. (2004) A hybrid heuristic approach to discrete multi-objective optimization of credit portfolios. *Computational Statistics & Data Analysis, 47*(2), 373-399. DOI: https://doi.org/10.1016/j.csda.2003.11.016.

Shannon, C. (1948). A mathematical theory of communication. *The Bell System Technical Journal, 27*(3), 379-423. DOI: https://doi.org/10.1002/j.1538-7305.1948.tb01338.x

Usta, I., & Kantar, Y. M. (2011) Mean-variance-skewness-entropy measures: A multi-objective approach for portfolio selection. *Entropy, 13*(1), 117-133. DOI: https://doi.org/10.3390/e13010117.

Yen, Y., & Yen, T. (2014). Solving norm constrained portfolio optimization via coordinate-wise descent algorithms. *Computational Statistics & Data Analysis, 76*, 737-759. DOI: https://doi.org/10.1016/j.csda.2013.07.010.